Addendum - Numerical results for setting with Time 1 uncertainty

This addendum to Caskey and Hughes (2012) provides numerical analysis for a setting in which there is residual uncertainty at Time 1 and the project selection problem involves risk-shifting. We restate the observation and propositions from the analytical model in the manuscript:\footnote{Proposition 2 places restrictions not present in Proposition 1 because the presence of renegotiation creates complications in determining whether debt value is increasing in maturity value.}

**Observation 1.** A necessary condition for a debt covenant to deter shareholders from choosing the low NPV project is that the creditors receive control rights for some \((v_2, s_1)\) such that \(M - R < s_1 < M < v_2\), referred to as the ‘essential region.’

**Proposition 1** (No renegotiation). Given parameters such that full creditor control would deter shareholders from choosing the low NPV project, then the minimum value (continuation value) covenant implements the baseline project with strictly lower expected inefficiency costs than the abandonment value (maximum value) covenant.

**Proposition 2** (Renegotiation). Given parameters such that full creditor control would deter shareholders from choosing the low NPV project, and assuming that maturity value \(M\) is decreasing in the covenant threshold \(k\), then the minimum value (continuation value) covenant can implement the baseline project with a strictly lower probability of renegotiation than the abandonment value (maximum value) covenant. Sufficient conditions for such orderings to exist in equilibrium are that \(s_0 = \mathbb{E}[s_1]\), the project has a positive expected payoff even absent the abandonment option \((\mathbb{E}[v_2] > s_0)\) and that the expected project return, cum abandonment option, is less than 100\% \((\mathbb{E}[\max\{v_2, s_2\}] < 2s_0)\).

Section I describes the setup. Section II provides numerical analysis for the case without renegotiation and Section III provides the analysis when renegotiation is allowed. The Appendix derives shareholders’ and creditors’ continuation preferences.

## I Setup

We reproduce the timeline in Figure A1. At Time 0, shareholders offer a take-it-or-leave-it debt contract to creditors. Because creditors are rational and competitive, the present value of expected payoffs to creditors under the contract must equal the loan proceeds. After the contract is signed, shareholders invest the borrowed funds in a project of their choosing. The project choice is non-contractible. At Time 1, information about the project’s values if continued and if abandoned becomes available to both parties after which a continuation decision is made. A debt covenant, if any, determines whether creditors rather than shareholders control that decision. Similar to technical default conditions, debt covenants are defined by the sets of continuation and abandonment values...
for which control over the continuation decision is transferred to creditors. Upon obtaining control, creditors learn which project has been selected and make their continuation decision based on that knowledge.

Time 0

- Firm raises $s_0$ using zero-coupon debt
- Debt covenant is set
- Shareholders choose project subsequent to issuing debt

Time 1

- Covenant threshold violation assessed
- If violated, creditors control continuation
- If project is abandoned, value $s_1$ is realized; creditors receive $\min\{e^{-r} M, s_1\}$ and shareholders receive $\max\{0, s_1 - e^{-r} M\}$

Time 2

- If project was continued at Time 1, value $v_2$ is realized; creditors receive $\min\{v_2, M\}$ and shareholders receive $\max\{0, v_2 - M\}$

Figure A1: Timeline

If the project is abandoned, creditors receive the lesser of the project’s abandonment value or the present value of the debt’s face amount. Shareholders receive any excess of the abandonment value over the present value of the debt’s face amount. If the project is continued, it yields a payoff at Time 2 and creditors receive the lesser of the project’s payoff or the face amount and shareholders receive any excess of the payoff over the face amount. We also consider a setting where the party holding control rights can renegotiate by making a take it or leave it offer.

Setup

Shareholders and creditors share the Time 0 prior that the abandonment value $s_1$ and continuation value $v_1$ are lognormal variables:

$$
\frac{\log(v_1/v_0)}{\log(s_1/s_0)} \sim \mathcal{N}\left(\mu_v - \frac{1}{2}\sigma_v^2, \mu_s - \frac{1}{2}\sigma_s^2 \rho\sigma_v\sigma_s, \sigma_s^2 \rho\sigma_v\sigma_s, \sigma_v^2 \right),
$$

where the parameters and starting points $v_0$ and $s_0$ are known at Time 0 and $s_0$ denotes the initial investment. The evolution of the continuation value from Time 1 to Time 2 is also lognormal with $\log(v_2/v_1) \sim \mathcal{N}\left(\mu_v - \frac{1}{2}\sigma_v^2, \sigma_v^2 \right)$. At Time 1, the abandonment value

\footnote{These priors are equivalent to assuming that continuation value (abandonment value) follows the process $dv_t = \nu_v \sigma_v dt + \sigma_v d\zeta_{vt}$, $ds_t = \mu_s \sigma_s dt + \sigma_s d\zeta_{st}$ where $\zeta_{vt}$ and $\zeta_{st}$ are correlated Brownian motions with $d\zeta_{vt} d\zeta_{st} = \rho dt$.}
s_1$ and expected continuation value $E_1[v_2] = e^{\mu v} v_1$ are realized. The zero-coupon debt’s maturity value is $M$.

### Continuation preferences

At the Time 1 continuation decision, the first-best choice is to continue whenever the abandonment value $s_1$ is less than the value $e^{-r} E_1[v_2] = e^{\mu v} v_1$ of continuing, implying the first-best rule to continue when $s_1 < e^{-r} E_1[v_2] \equiv \hat{v}$. Shareholders prefer to continue when the value $e^{-r} E_1[\max\{0, v_2 - M\}]$ exceeds the value $\max\{0, s_1 - e^{-r} M\}$ from abandoning. This is equivalent to continuing when $s_1 < e^{-r} E_1[\max\{v_2, M\}] \equiv s_e$. Creditors prefer to continue if the value of continuing $e^{-r} E_1[\min\{v_2, M\}]$ exceeds the value $\min\{s_1, e^{-r} M\}$ of abandoning. This is equivalent to continuing when $s_1 < e^{-r} E_1[\min\{v_2, M\}] \equiv s_d$.

Figure A2, Panel A depicts the shareholder and creditor continuation preferences for realizations of the continuation value $\hat{v}$ and abandonment value $s_1$. Creditors’ limited upside makes them risk-averse and more willing to abandon, while shareholders’ limited downside makes them more willing to bear risk by continuing. Hence, the shareholders will continue unless the abandonment value $s_1$ exceeds the higher threshold $s_e$ than the $s_d$ threshold for which creditors will abandon. Neither shareholders nor creditors follow the first-best continuation rule in all cases so that the allocation of control rights, via a debt covenant, determines extent to which the continuation decision will be made in an efficient manner.

### Project selection

As in the classic asset substitution problem, shareholders have a convex claim against firm value that provides incentive to pursue a riskier, lower NPV project. We represent high risk/low NPV projects by increasing the volatility $\sigma_v$ of the continuation value while reducing its drift $\mu_v$. Because the expected value of continuation $E[v_2] = e^{2\mu_v} v_0$, the reduction in drift represents a lower NPV investment. If the sacrifice in NPV is not too great relative to the higher risk, the shareholders’ loss of value from the lower drift will be compensated by their benefit from higher volatility.

Creditors have rational expectations and anticipate this behavior, incorporating it into the required maturity value $M$ determined at Time 0. As a result, shareholders ultimately bear the cost of pursuing the low NPV project if they cannot help doing so, ex post. While creditors observe the chosen project if they gain control over the continuation decision,

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3We use the notation $E_1[\cdot] \equiv E[\cdot|v_1, s_1]$.

4This condition derives first from the fact that $\max\{0, x - y\} = \max\{y, x\} - y$ and then from the fact that $E_1[\max\{v_2, M\}] \geq M$, with equality only if $v_2$ is known at Time 1. Thus, if the abandonment value $s_1$ exceeds $e^{-r} E_1[\max\{v_2, M\}]$, then it necessarily exceeds $e^{-r} M$ so that $\max\{s_1, e^{-r} M\} = s_1 > e^{-r} E_1[\max\{v_2, M\}]$.

5This condition derives first from the fact that $E_1[\min\{v_2, M\}] \leq M$, with equality only if $v_2$ is known at Time 1. Thus, $\min\{s_1, e^{-r} M\} < e^{-r} E_1[\min\{v_2, M\}]$ if and only if $s_1 < e^{-r} E_1[\min\{v_2, M\}] < e^{-r} M$.

6Volatility, by itself, increases firm value on account of the abandonment option. Lowering NPV therefore requires that the inferior project have a sufficiently low drift to compensate for the value created by higher volatility.
Figure A2: Continuation preferences

Figure A2 illustrates shareholders’ and creditors’ Time 1 continuation preferences as a function of the expected value of continuing $\hat{\nu} = e^{-r}E_1[v_2] = e^{\mu - r}v_1$ and abandonment value $s_1$ given a debt maturity value $M$. The diagonal line $\hat{\nu}$ represents the first-best rule of continuing when $\hat{\nu} > s_1$. The line $s_e$ represents shareholders’ preference to continue when $e^{-r}E_1[\max\{0, v_2 - M\}] > \min\{0, s_1 - e^{-r}M\}$. The line $s_d$ represents creditors’ preference to continue when $e^{-r}E_1[\min\{v_2, M\}] > \min\{s_1, e^{-r}M\}$. Panel B illustrates the upward (downward) shift in thresholds for shareholders (creditors) as a result of increasing the risk of continuing the project.

At the Time 1 continuation decision, the higher risk of the low NPV project increases the shareholders’ payoff from continuing while it reduces the creditors’ payoff from continuing. Figure A2, Panel B illustrates these shifts in continuation preferences. The lowest abandonment value for which shareholders continue, $s_e$, shifts upward while the lowest abandonment value for which creditors abandon, $s_d$, shifts downward. The shift in creditor continuation preferences plays a crucial role in the ability of debt covenants to deter shareholders from pursuing low NPV projects. Specifically, the transfer of control rights to creditors makes it more likely that risky projects will be abandoned. This, in turn, makes the high risk/low NPV projects less attractive to shareholders. Granting control rights to creditors has a cost, however, because in some cases creditors also prefer to abandon even a good project when it should instead be continued. Our analysis

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7 This assumption is consistent with the non-contractibility of the project choice and with the legal environment that grants significant leeway to corporate managers under the business judgment rule.
II Numerical Results for Setting without Renegotiation

Given uncertainty at Time 1 and full control at the continuation stage, shareholders have an incentive to substitute higher variance, lower NPV projects in place of the project that maximizes firm value. We show that Proposition 1’s prediction of the dominance of covenants based on continuation value (minimum value) accounting measure over the maximum value (abandonment value) measure continues to hold in this setting.

In contrast to the setting without uncertainty at Time 1, nowhere are either shareholders’ or creditors’ continuation preferences exactly aligned with the first-best rule. Figure A3, Panel A illustrates where inefficient continuation decisions occur, depending on which party controls the decision. The dashing indicates regions where shareholders inefficiently continue (creditors inefficiently abandon). The shareholders’ (creditors’) continuation preferences are more closely aligned with the first-best rule in the upper right (lower left) quadrants of the figure; however, uncertainty pushes the preferences away from first-best because of the shareholders’ (creditors’) willingness (reluctance) to bear risk. The lightly shaded regions Figure A3, Panel A denote where Time 1 uncertainty creates additional inefficiencies by shifting shareholders’ and creditors’ continuation preferences.

Figure A3, Panel B illustrates the analogue to the ‘essential region’ from Observation 1 where creditors continue the baseline project but abandon the low NPV project. The higher risk from the low NPV project shifts the creditors’ continuation preference downward and the essential region lies between the creditors’ continuation preference curves for the baseline and low NPV projects. Because continuation always has value to shareholders in this setting, the essential region is present in the lower left quadrant whereas it was not in the setting of the manuscript where there is no Time 1 uncertainty. As a result, the continuation value and maximum value covenants have some value for deterring low NPV projects even when set with low thresholds, whereas they required default thresholds greater than the maturity value $M$ to deter the low NPV project in the manuscript’s analysis.

Consistent with the results of the manuscript, there is no clear dominance relation between covenants based on the continuation value and minimum value covenants. Both the minimum value and abandonment value covenants allocate control to creditors for a greater portion of the essential region than the continuation value and maximum value covenants. This comes at the cost of inefficiencies in the continuation decision, however. Figure A4, Panel A compares the inefficient interim decisions resulting from continuation value and minimum value covenants. The allocation of control to creditors in the lower right quadrant dashed region results in additional inefficient continuation decisions with the minimum value covenant versus a continuation value covenant.
Figure A3, Panel A illustrates the \((\hat{v}, s_1)\) realizations for which shareholders (creditors) will inefficiently continue (abandon) the baseline project if they control the continuation decision. The light gray shaded area denotes the added areas where inefficient decisions occur on account of uncertainty at Time 1 about the value of continuing. Panel B illustrates how creditor control affects the continuation of the baseline project versus a high risk/low NPV project. The dark (light) gray area denotes \((\hat{v}, s_1)\) realizations for which creditors abandon (continue) either project. The middle area denotes where creditors continue the baseline project but abandon a high risk/low NPV project, and corresponds to the essential region from Observation 1.

Figure A4, Panel B illustrates the maximum allocation of control to creditors without creating any inefficient continuation decisions. Similar to measure discussed in Caskey and Hughes (2012, pp. 501-503), such a covenant allocates control to shareholders only where creditors make an inefficient choice to abandon. In other words, shareholders obtain control if and only if \(\hat{v} > s_1 > \bar{s}_d\). Also, the same measure of asset value can be used in this case – namely, reporting \(s_1\) when it is optimal to continue \((\hat{v} > s_1)\) and reporting \(\min\{\hat{v}, s_0\}\) when it is optimal to abandon. In contrast to the maximum-creditor-control covenant in the manuscript, the threshold for default must vary. In particular, it must be the creditors’ continuation threshold \(\bar{s}_d\). While it is uncommon to report such a moving target, it is possible to recast the covenant as a net worth covenant where liabilities are reported at fair value. In order to see this, first note that if this covenant is sufficient to deter a low NPV project, then investors will correctly infer that the firm will pursue the baseline project. Because this covenant implements the first-best continuation policy, the project is continued for \(\hat{v} > s_1\) in which case the debt value will
The diagonal dashed (shaded) regions in Figure A4, Panel A illustrate the set of $(\hat{v}, s_1)$ for which, absent renegotiation, shareholders inefficiently continue and creditors inefficiently abandon under a continuation value (minimum value) covenant with technical default threshold $k$. The threshold $k < e^{-r}M$ is set to minimize the cost of inefficient continuation and abandonment decisions with the continuation value covenant (the maturity value and default threshold for the corresponding minimum value covenant are nearly identical). The dashed regions in Panel B illustrate the largest set of $(\hat{v}, s_1)$ for which creditors can have control without yielding inefficient continuation decisions. The shaded region in Panel B is the set of $(\hat{v}, s_1)$ realizations for which creditors continue the baseline project but abandon a high risk/low NPV project.

$$e^{-r}E_1[\min\{v_2, M\}] = \bar{s}_d.$$ If the project is abandoned for $\hat{v} < s_1$ then the debt value will be $\min\{s_1, e^{-r}M\}$. In conjunction with an asset value determined in this manner and a zero net worth threshold for technical default, we then have a net worth that yields
The desired result that shareholders only obtain control when $\hat{v} > s_1 > s_d$:  

$$
\text{Reported equity value if continued (} \hat{v} > s_1 \text{)} = s_1 - s_d > 0 \iff \hat{v} > s_1 > s_d.
$$

$$
\text{Reported equity value if abandoned (} \hat{v} < s_1 \text{)} = \min\{\hat{v}, s_0\} - \min\{s_1, e^{-r} M\} < 0.
$$

The relative advantages of the continuation value covenant with respect to continuation and of the minimum value-covenant with respect to project selection are evident in the results in this analysis. Although unconventional, the covenant based on historical cost and fair values that provides maximum control to creditors outperforms both of these.

Parameterization of alternative projects

We model continuation value $v_t$ and abandonment value $s_t$ as lognormal variables as in (1). We set the risk-free rate to 0.05 as in Décamps and Faure-Grimaud (2002) which is between 0.045 used by Goldstein et al. (2001) and 0.06 used by Leland (1998). We denote the baseline project as Project A and set its drift parameters for continuation value $\mu_v$ and abandonment value $\mu_s$ at double and half the risk free rate, respectively. The continuation value’s diffusion parameter $\sigma_v$ is 0.40 as in Décamps and Faure-Grimaud (2002), while the abandonment value’s diffusion parameter $\sigma_s$ is half at 0.20, consistent with less volatile values in abandonment. The correlation is set at 0.40 based on the correlation between log asset and log market value returns of Compustat firms. The initial continuation and abandonment values are set at 100. These parameters imply that the ex ante value of the project, inclusive of the abandonment option, is 120.1, which is equivalent to a 20% return.

We consider a menu of low NPV projects, $S_1$, $S_2$, $S_3$, and $S_4$, that model the asset substitution problem of high-risk, low NPV projects. Accordingly, the volatility for all four alternative projects is set at 0.70, which is higher than Project A’s volatility of 0.4. Recall from Figure A2, Panel B that the higher risk shifts creditors’ continuation preferences downward, reflecting their greater likelihood of abandoning risky projects. We modeled this force in Caskey and Hughes (2012) using the reduction $R$ in creditors’ payoff when shareholders pursued the low NPV project. The expected continuation value of a given project is $e^{-2r} E[v_2] = e^{2(\mu_v - r)} v_0$, so that the drift parameter determines their value. We, accordingly, assign lower drift parameters to the low NPV Projects $S_1$, $S_2$, $S_3$, and $S_4$ of -0.05, -0.03, -0.003, and 0.005, respectively. These parameters imply that

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8The inequality in the abandonment case follows because $\min\{\hat{v}, s_0\} \leq \hat{v} < s_1$ and $\min\{\hat{v}, s_0\} \leq s_0 < e^{-r} M$. We note, however, that in some cases in a setting that allows for renegotiation it is possible that $s_0 > e^{-r} M$, which could arise if creditors give generous credit terms in expectation of extracting gains from successful projects. In such cases, $s_0$ in the expression would have to be replaced with some number less than $e^{-r} M$ in order for the covenant to both provide as much control to creditors as possible without creating inefficient continuation decisions.
the first-best ex ante values of the projects, inclusive of the abandonment option, are 113.2, 115.0, 117.6, and 118.5, respectively. Hence, in the absence of conflicts between shareholders and creditors, Project A would be preferred to all substitutes and Project $S_4$ would be the best of the substitutes. Table A1 summarizes project parameters.

Table A1: Project parameters

Table A1 displays the parameters of the project choices we consider the asset substitution problem.

<table>
<thead>
<tr>
<th></th>
<th>Project</th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>$S_3$</td>
<td>$S_4$</td>
</tr>
<tr>
<td>Continuation value</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Initial value $v_0$</td>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<td>Drift $\mu_v$</td>
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<td>-0.050</td>
<td>-0.030</td>
<td>-0.003</td>
<td>0.005</td>
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<td>Volatility $\sigma_v$</td>
<td>0.40</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
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<tr>
<td>Abandonment value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial value $s_0$</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Drift $\mu_s$</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
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<td>Volatility $\sigma_s$</td>
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<td>0.20</td>
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<tr>
<td>Correlation $\rho_{vs}$</td>
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<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
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<tr>
<td>Risk-free rate</td>
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<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Value with first-best implementation</td>
<td>120.1</td>
<td>113.2</td>
<td>115</td>
<td>117.6</td>
<td>118.5</td>
</tr>
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</table>

We denote the Time 1 expected continuation value by $\hat{v} = e^{-r} E_1[v^2] = e^{\mu_v - r} v_1$, where $\mu_v$ depends on the choice of project. The maximum value and minimum value covenants are based on reported measures $\max\{\hat{v}, s_1\}$ and $\min\{\hat{v}, s_1\}$ respectively. As a benchmark, we first set thresholds for each of these covenants so that the combination of threshold $k$ and resulting maturity value $M$ minimize the loss in value from inefficient continuation and abandonment decisions assuming that the firm pursues the baseline Project A. The first column in Table A2 presents the loss in value under the different covenants along with the covenant threshold and maturity value that characterize the debt contract.

Comparisons of covenant performance

Figure A5 plots the shareholders’ indifference curves over the parameters $(\mu_v, \sigma_v)$ given the baseline contracts for each accounting measure. In addition, we plot the indifference curves for a contract that gives shareholders (creditors) control for when it is optimal to continue (abandon) and thus entails no loss from inefficient continuation decisions. Figure A5 also displays the menu of low NPV projects. The shareholders’ expected payoff is increasing as we move to the right (increasing expected return) and up (increasing volatility). The covenant (2) that incorporates historical costs deters all four projects without any costs
Table A2: Covenants and equity values with potential asset substitution and no renegotiation

Table A2 illustrates the parameters of debt contracts that minimize the expected costs of inefficient continuation and abandonment subject to the constraint that shareholders prefer Project A to the Project S_i listed in the columns of the table.

<table>
<thead>
<tr>
<th>Optimized Contract implements A vs. S_i</th>
<th>Project value</th>
<th>Value loss versus A</th>
<th>Full creditor control</th>
<th>Abandonment value s_1 covenant</th>
<th>Minimum value min{s_1, ̂v} covenant</th>
<th>Maximum value max{s_1, ̂v} covenant</th>
</tr>
</thead>
<tbody>
<tr>
<td>for A</td>
<td>120.1</td>
<td>6.9</td>
<td>136.4</td>
<td>135.1</td>
<td>132.2</td>
<td>135.1</td>
</tr>
<tr>
<td>S_1</td>
<td>113.2</td>
<td>5.1</td>
<td>136.4</td>
<td>135.1</td>
<td>132.2</td>
<td>135.1</td>
</tr>
<tr>
<td>S_2</td>
<td>115</td>
<td>2.5</td>
<td>136.4</td>
<td>135.1</td>
<td>132.2</td>
<td>135.1</td>
</tr>
<tr>
<td>S_3</td>
<td>117.6</td>
<td>1.6</td>
<td>135.8</td>
<td>131.8</td>
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<tr>
<td>S_4</td>
<td>118.5</td>
<td></td>
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<td>129</td>
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<td>129</td>
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<tr>
<td>Debt face value</td>
<td></td>
<td></td>
<td>136.4</td>
<td>135.1</td>
<td>132.2</td>
<td>135.1</td>
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<tr>
<td>Covenant threshold</td>
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<td>123.7</td>
<td>124.3</td>
<td>126.5</td>
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<td>Inefficiency costs</td>
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<td>2.25</td>
<td>2.25</td>
<td>2.26</td>
<td>2.49</td>
</tr>
<tr>
<td>Continuation value (e^{-r}E[v_2</td>
<td>v_1]=\hat{v}) covenant</td>
<td></td>
<td></td>
<td>115</td>
<td>115</td>
<td>242.4</td>
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<td>Debt face value</td>
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<td>135.1</td>
<td>135.1</td>
<td>132.7</td>
<td>121.6</td>
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<td>Covenant threshold</td>
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<td>107.4</td>
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<tr>
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<td>0.4</td>
<td>4.73</td>
<td>5.55</td>
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<td>Minimum value min{s_1, ̂v} covenant</td>
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<tr>
<td>Debt face value</td>
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<tr>
<td>Covenant threshold</td>
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<tr>
<td>Inefficiency costs</td>
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</table>

of inefficiencies and, furthermore, lies quite close to the thick dashed line that denotes \((\mu_v, \sigma_v)\) combinations that yield the same first-best NPV as Project A.

When considering the ability of the other covenants to deter low NPV projects, it is evident from Figure A5 that any of the covenants will deter S_1, the least attractive low NPV project, when set at the value that minimizes losses from inefficient continuation decisions. The second column in Table A2 reports the contract maturities and covenant thresholds that minimize losses subject to deterring Project S_1. While all four covenants implement Project A in lieu of S_1, the continuation value covenant does so at the lowest
(A) Set of \((\mu_v, \sigma_v)\) yielding NPV of 20.097 equal to that of the baseline Project \(A\)

(B) Shareholder preferences for projects under different covenants

Figure A5: Project selection in setting without renegotiation

Figure A5 plots the set of expected return \(\mu_v\) and volatility \(\sigma_v\) for the value of continuing the project, which represent project choices. The thick dashed line in Panel A plots the set of \((\mu_v, \sigma_v)\) that yield an NPV equal to the 20.097 of the baseline Project \(A\) and the set of alternative low NPV projects \(S_1, S_2, S_3\) and \(S_4\). Panel B plots shareholder indifference curves over \((\mu_v, \sigma_v)\) under different covenants where the debt contracts minimize the expected costs of inefficient continuation decisions assuming that the firm pursues Project \(A\).

Moving to the next most attractive low NPV project, \(S_2\), the abandonment and minimum value covenants continue to implement Project \(A\) while the continuation and maximum value covenants must be altered. The abandonment and minimum value covenants' superior ability to deter \(S_2\) follows from their greater allocation of control rights in the “essential region” where creditors abandon the low NPV project but continue the baseline project. While these covenants deter \(S_2\) with no alteration, a slight modification allows the continuation value covenant to deter \(S_2\), as well, and at a lower cost. The abandonment and minimum value covenants over-protect since they deter not only \(S_2\), but other projects. This protection comes at the cost of giving creditors control where they inefficiently abandon. As is apparent from the comparison of the continuation value and minimum value covenants in Figure A4, Panel A, the continuation value covenant yields cost with losses of only 0.4 versus Project \(A\)’s first-best value of 120.1.
fewer inefficient abandonments. This advantage persists after the minor adjustment in the threshold needed to deter $S_2$ using a continuation value covenant.

The low NPV Projects $S_3$ and $S_4$ are relatively more attractive to shareholders than Projects $S_1$ and $S_2$, making the project selection problem more severe. None of the covenants deter these projects when set in order to merely minimize losses from inefficient continuation decisions. When the contracts are altered to deter these projects, the minimum value covenant performs best with losses of 1.01 and 1.51 in order to deter projects $S_3$ and $S_4$, respectively. Given the severity of the project selection problem, the greater allocation of control rights to creditors under the minimum value covenant, even with low covenant thresholds, becomes relatively more important.

Deterring $S_3$ and $S_4$ with the continuation value covenant requires giving creditors essentially total control of the continuation decision. Deterring Project $S_3$, for example, entails costs of 4.73. The resulting expected firm value is the first-best value of 120.1 less the costs of 4.73, or 115.37. Since the value of Project $S_3$ with first-best implementation is 117.6, the firm would be better off implementing the low NPV project than trying to implement Project $A$ using a covenant based on continuation value.

In all of the examples, the continuation value (minimum value) covenant dominates maximum value (abandonment value), consistent with the prediction of Proposition 1. This suggests that the intuition we developed in the manuscript, where we exclude Time 1 uncertainty, carries over to the more general setting. Given these dominance relations, the covenant choice among the set we consider is reduced to a choice between the continuation value and minimum value covenants and the covenant that incorporates fair value and historical cost. The continuation value covenant appears to be more effective in minimizing continuation inefficiencies while the minimum value covenant appears to be more effective in deterring low NPV projects. When low NPV projects are not very tempting to managers, the efficiency of contracts is primarily driven by minimizing continuation inefficiencies and the continuation value covenant performs better, consistent with Gigler et al.’s (2009) prediction that conservative accounting destroys value in settings without project selection. When low NPV projects become more tempting, project selection concerns dominate and the minimum value covenant performs better. Which dominates depends on the relative importance of the two sources of shareholder/creditor conflict. In no case does the FASB’s “highest and best use” measure appear to provide an effective basis for debt contracts.

III Numerical Results for Setting with Renegotiation

This section provides numerical results analogous to Section II with the exception that the contract can be renegotiated at the interim stage. The continuation preferences and parameters are identical to that in Section II and as given in Table A1. We measure the effectiveness of accounting measures by the probability of renegotiation, which we view as a proxy for expected renegotiation costs.

Figure A6 reproduces the indifference curves from Figure A5, Panel B for the case where the contract can be renegotiated. The contracts for each covenant type are set with
the covenant threshold and maturity value that minimize the probability of renegotiation and creditors breakeven. The numerical results are consistent with Proposition 2 and the numerical results of Section II where we evaluate covenants based on the costs of inefficient continuation decisions. In particular, the continuation value (minimum value) covenant yields a lower probability of renegotiation than the maximum value (abandonment value) covenant.

Table A3 presents the contract parameters, maturity value and covenant threshold, set to minimize the probability of renegotiation subject to making shareholders favor the baseline Project A. Consistent with Proposition 2, the minimum value (continuation value) covenant is able to dissuade each project with a covenant having a strictly lower probability of renegotiation than with an abandonment value (maximum value covenant). Also, consistent with the numerical results in Section II, the maximum value covenant

![Figure A6: Project selection in setting with renegotiation](image)

Figure A6 plots the shareholder indifference curves over \((\mu_v, \sigma_v)\) under different covenants where the debt contracts minimize the expected costs of inefficient continuation decisions assuming that the firm pursues Project A.
implements Project $A$ with a lower probability of renegotiation than the continuation value covenant when asset substitution is more tempting to shareholders. As in the analysis of Section II, the fair value/historical cost covenant given in (2) provides a greater deterrent to asset substitution than any of the fair value covenants we consider.

Table A3: Covenants and equity values with potential asset substitution with renegotiation

Table A3 illustrates the parameters of debt contracts that minimize the probability of renegotiation subject to the constraint that shareholders prefer Project $A$ to the Project $S_i$ listed in the columns of the table.

<table>
<thead>
<tr>
<th>Optimized</th>
<th>Contract implements $A$ vs. $S_i$ for $A$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project value</td>
<td>120.1</td>
<td>113.2</td>
<td>115</td>
<td>117.6</td>
<td>118.5</td>
</tr>
<tr>
<td>Full creditor control</td>
<td>Debt face value</td>
<td>91.2</td>
<td>91.2</td>
<td>91.2</td>
<td>91.2</td>
</tr>
<tr>
<td></td>
<td>P(Renegotiation; $A$)</td>
<td>0.497</td>
<td>0.497</td>
<td>0.497</td>
<td>0.497</td>
</tr>
<tr>
<td>Abandonment value $s_1$ covenant</td>
<td>Debt face value</td>
<td>139</td>
<td>136.7</td>
<td>133.5</td>
<td>118.9</td>
</tr>
<tr>
<td></td>
<td>Covenant threshold</td>
<td>118.2</td>
<td>119.4</td>
<td>120.6</td>
<td>119.6</td>
</tr>
<tr>
<td></td>
<td>P(Renegotiation; $A$)</td>
<td>0.159</td>
<td>0.160</td>
<td>0.165</td>
<td>0.222</td>
</tr>
<tr>
<td>Continuation value $e^{-r}E[v_2</td>
<td>v_1] = \hat{v}$ covenant</td>
<td>Debt face value</td>
<td>133.7</td>
<td>133.7</td>
<td>133.1</td>
</tr>
<tr>
<td></td>
<td>Covenant threshold</td>
<td>111.4</td>
<td>111.4</td>
<td>114.1</td>
<td>443.9</td>
</tr>
<tr>
<td></td>
<td>P(Renegotiation; $A$)</td>
<td>0.076</td>
<td>0.076</td>
<td>0.077</td>
<td>0.496</td>
</tr>
<tr>
<td>Minimum value $\min{s_1, \hat{v}}$ covenant</td>
<td>Debt face value</td>
<td>135</td>
<td>135</td>
<td>135</td>
<td>130.5</td>
</tr>
<tr>
<td></td>
<td>Covenant threshold</td>
<td>105</td>
<td>105</td>
<td>105</td>
<td>114.4</td>
</tr>
<tr>
<td></td>
<td>P(Renegotiation; $A$)</td>
<td>0.095</td>
<td>0.095</td>
<td>0.095</td>
<td>0.115</td>
</tr>
<tr>
<td>Maximum value $\max{s_1, \hat{v}}$ covenant</td>
<td>Debt face value</td>
<td>131.3</td>
<td>131.3</td>
<td>130.6</td>
<td>91.2</td>
</tr>
<tr>
<td></td>
<td>Covenant threshold</td>
<td>128.1</td>
<td>128.1</td>
<td>129.4</td>
<td>443.9</td>
</tr>
<tr>
<td></td>
<td>P(Renegotiation; $A$)</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
<td>0.496</td>
</tr>
</tbody>
</table>

IV Summary

This addendum to Caskey and Hughes (2012) examines whether the covenant efficacy predictions from the manuscript carry over to a setting with explicit risk shifting and
residual uncertainty at the Time 1 continuation decision. The main difference between the two settings is that none of the fair value covenants can completely avoid inefficiencies at the Time 1 continuation decision. Consistent with the predictions in the manuscript, the conservative minimum value measure best balances the deterrence of asset substitution and implementing efficient continuation decisions when asset substitution poses a greater attraction to shareholders. However, ignoring the need to deter asset substitution, the continuation value covenant better implements efficient continuation decisions, suggesting that the conservative measure detracts from firm value unless there is an asset substitution problem to mitigate. We also examine a net worth measure that values debt at fair value and assets using a hybrid of fair value and historical cost. This covenant represents the greatest control that can be given to creditors while maintaining zero costs of inefficient continuation decisions, and better deters asset substitution than any of the fair value covenants.
### Derivation of continuation preferences

The overall value of continuing at Time 1 is:

$$e^{-r} E[v_2|v_1, s_1] = e^{-r} E[v_2|v_1] = e^{\mu_v - r} v_1 \equiv s_*.$$  

(B.1)

Shareholders prefer to continue when $\max\{0, s_1 - e^{-r} M\} < e^{-r} E[\max\{0, v_2 - M\}|v_1]$, which holds if and only if $s_1$ is below the following threshold where $\Phi(\cdot)$ denotes the standard normal distribution:

$$s_e \equiv e^{-r} E[\max\{0, v_2, M\}|v_1] = e^{\mu_v - r} v_1 \Phi\left(\frac{\log(v_1/M) + \mu_v + \sigma_v^2/2}{\sigma_v}\right) + e^{-r} M \Phi\left(-\frac{\log(v_1/M) + \mu_v - \sigma_v^2/2}{\sigma_v}\right).$$  

(B.2)

The function $s_e$ asymptotically approaches $e^{-r} M$ as $v_1 \to 0$ and approaches $s_*$ as $v_1 \to \infty$.

Creditors prefer to continue when $\min\{s_1, e^{-r} M\} < e^{-r} M E[\min\{v_2, M\}|v_1]$, which holds if and only if $s_1$ is below the threshold:

$$s_d \equiv e^{-r} E[\min\{v_2, M\}|v_1] = e^{\mu_v - r} v_1 \Phi\left(-\frac{\log(v_1/M) + \mu_v + \sigma_v^2/2}{\sigma_v}\right) + e^{-r} M \Phi\left(-\frac{\log(v_1/M) + \mu_v - \sigma_v^2/2}{\sigma_v}\right).$$  

(B.3)

The function $s_d$ asymptotically approaches $s_*$ as $v_1 \to 0$ and approaches $e^{-r} M$ as $v_1 \to \infty$.

Direct comparison of expressions (B.1) through (B.3) shows that $s_d < s_* < s_e$ for all $v_1$. The formulas for the Black-Scholes option value's sensitivity to volatility give the following where $\phi(\cdot)$ denotes the density for the standard normal distribution:

$$\frac{\partial s_e}{\partial \sigma_v} = -\frac{\partial s_d}{\partial \sigma_v} = e^{\mu_v - r} v_1 \phi\left(\frac{\log(v_1/M) + \mu_v + \sigma_v^2/2}{\sigma_v}\right) > 0,$$  

which shows that increases in risk make shareholders more likely to continue and make creditors less likely to continue.

#### References


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9Expression (B.2) follows from adding $e^{-r} M$ to the Black-Scholes value of a call option and using the symmetry of the normal distribution, $1 - \Phi(x) = \Phi(-x)$. 

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